Misperceived vs Unanticipated Money: A Synthesis*

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Abstract

The role played by unperceived vs unanticipated money in the monetary transmission mechanism was the key issue in the macroeconomic debates that followed the introduction of rational expectations in the 70s. A similar debate seems to have re-emerged involving the sticky price, unanticipated shocks (the New Keynesian) model and the flexible price, misperceived shocks (the sticky information) model. We contribute to this debate by establishing three things. First, properly constructed misperceived money is quantitatively substantial and also matters significantly for economic activity. Second, *both* misperceived and unanticipated money, together with sticky prices are essential if a model is to generate plausible inflation dynamics following a monetary policy shock. And third, the degree of mis-perception (information stickiness) required for this is small.

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Introduction

The introduction of rational expectations in macroeconomics by Robert Lucas (Lucas, 1977) marked a turning point in the evolution of macroeconomic in the twentieth century. Lucas' model provided a coherent, micro-founded framework for the analysis of business cycles and, in particular, for studying the role played by money in macroeconomic fluctuations. The key element in his flexible price, monetary business cycle model is the distinction between perceived and misperceived monetary aggregates. Fluctuations in the supply of money matter for economic activity only to the extent that they are not perceived.

The Keynesian camp responded by adopting rational expectations too, but imbedded them in models with fixed nominal prices or wages (Fischer, 1977, Gray, 1976, Taylor, 1980). The fixity of prices made another type of change in the money supply, namely unanticipated, to play the key role in generating business cycles. In the rational expectations, imperfect information, Keynesian model, prices were set in relation to expectations of future money (inflation) growth. Consequently, movements in the actual supply of money that differed from those that had been anticipated affected relative prices and induced movements in aggregate economic activity.

A key macroeconomic debate of the second half of the seventies and most of the eighties thus involved the concepts of anticipated vs unanticipated and perceived vs misperceived money. The empirical evidence studied in the context of that debate favored the unanticipated money version. For instance, Barro and Rush, 1980, found that unanticipated money growth mattered for economic activity (while anticipated did not). Using the difference between preliminary and revised monetary data to model mis-perceptions, Barro and Hercowitz, 1981, and Boschen and Grossman, 1982, found that mis-perceived money did not matter for the business cycle.

These findings exerted great influence on the subsequent evolution of macroeconomic theory. Proponents of flexible price models abandoned en mass Phillips curve models and migrated to the RBC model. Proponents of fixed price models continued refining the Keynesian paradigm. Their research program adopted many features of the RBC model and culminated in the development of the New Keynesian (NK) model[1], which has become the leading monetary model of our days. Like its predecessor, it relies on unanticipated monetary shocks and sticky prices in order to generate monetary non-neutrality.

The last few years have witnessed the emergence of an important rival to the NK model, namely, the sticky information or inattentive agents model[2]. This model discounts the importance of nominal stickiness and emphasizes unperceived shocks as the main source of monetary non-

[1]Christiano et al, 2005, –henceforth CEE– Clarida, Gali and Gertler, 1999, Goodfriend and King, 1997, Woodford, 2003.

[2]Sims, 2003, Mankiw and Reis, 2002, 2006.

neutrality. Given these features, the sticky price model can be viewed as representing –at least in the domain of monetary business cycles– a simpler version of the model of Lucas[3]. In particular, while the model of Lucas involves a standard signal extraction problem, the sticky information model takes a short cut on learning by assuming that the signal extraction is of a special type: A fraction of the agents becomes periodically fully informed, while the rest has imperfect knowledge of the true state of the economy[4].

It thus seems that, while the specifics of the main monetary models of the business cycle may have changed, the key debate regarding the underlying cause of non-neutrality remains the same. Namely, whether it is mis-perceived or unanticipated money that makes money matter for real economic activity.

The objective of this paper is to contribute to this debate in two ways. First, by revisiting the issue of whether misperceived and unanticipated money matter empirically, an issue that has been left dormant for a long period. Using information from real time data (from the Philadelphia FED) we establish several facts: a) Standard measures of misperceived money –constructed from the initial release and the subsequent revisions of the money stock– have the properties required for testing imperfect information, rational expectations theories in the post 1982 but not in the pre 1982 period. In particular, relying on the approach of Mankiw et al., 1984, we show that for the post 1982 period, the preliminary announcement of the money stock is simply the true variable measured with error (classical errors-in-variables). This provides justification for our signal extraction formulation of the learning problem. And that the money stock revisions cannot be predicted based on information that is available at the time of the initial release. Hence, the standard measure of misperceived money (the difference between preliminary and revised releases) is legitimate as it does not contain anticipated components. But this is not true in the pre 1982 period, as money revisions during that period contain predictable elements[5]. c) Both misperceived money (the difference between preliminary and revised releases) and unanticipated money growth shocks are quantitatively substantial, with the former being more than half as volatile as the latter. d) Both of them matter for economic activity. Misperceived money –conventionally constructed– only matters in the post great inflation period, that is, post 1982.

The finding that the standard measure of misperceived money, namely, revised minus the initial release, does not matter for economic activity in the pre 1982 period is consistent with the results

[3]Given the failure of earlier empirical work to find a role for misperceived money, it is understandable that the sticky information, inattentive agents models have shunned away from dealing explicitly with this issue.

[4]We suspect that the sticky information structure can be explicitly formulated as a constrained case of a signal extraction problem.

[5]Similar results for this period regarding the properties of money stock revisions are found in the small literature that followed the Mankiw et al. 2004 (see, Mork, 1990). Note also that there is a related literature on the properties of *other* data revisions (such as Faust et al. 2005 who rely on the Mankiw et al. methodology or Aruoba, 2007, who uses a somewhat different methodology) which however, does not consider money revisions, the key variable in the present paper.

of Barro and Hercowitz, 1981, and Boschen and Grossman, 1982. But as Mankiw et al., 1984, argue in their evaluation of the Barro-Hercowitz and Boschen-Grossman results, the existence of predictability in the money stock revisions may introduce an anticipated component into the measure of misperceived money –to the extent that such predictable components are not filtered out– and this can bias the effect of "misperceived" money on economic activity towards zero. We confirm this assertion by purging the predictable elements from the money revisions. We find that properly constructed misperceived money matters for economic activity also in the pre 1982 period.

Our second objective is to work out the implications of misperceived and unanticipated money in a standard NK model that has been augmented to include a signal extraction problem a la Lucas[6]. We establish that both aspects of money growth are *sine qua non* for the ability of the NK model to have satisfactory performance along its most crucial –and often criticized– dimension. Namely, inflation inertia. Monetary mis-perceptions play a critical role in muting the effect of a monetary surprise and thus generating a sluggish initial response of inflation to a monetary shock. Then the sticky prices together with the various real rigidities kick in and allow the model to produce a hump shaped response of inflation even under fully rational expectations. All of these elements, namely, misperceived money, unanticipated money and price stickiness are required in order for the model to produce empirically plausible inflation dynamics following a monetary policy shock under *realistic* informational, nominal and real frictions. In particular, we find that the amount of misperceptions (or information stickiness) required quite small and within the range of empirical estimates.

The remaining of the paper is organized as follows. Section 1 presents the empirical evidence. Section 2 presents the model while Section 3 discusses its empirical properties.

1 Misperceived and Unanticipated Money

The early vintage of the imperfect information, rational expectations, flexible price models required that the agents did not observe any of the nominal aggregates. But it was quickly realized that it strained credibility to assume that monetary aggregates were not observable at all, or, that they were so but only with substantial time lags. King, 1981, offered a plausible alternative, under which information on monetary aggregates was assumed to be readily available but observations of the current or recent monetary data (the preliminary figures) were ridden with measurement error. This error was only gradually corrected through subsequent data revi-

[6]Bomfim, 2001, Aruoba, 2004 examine the effects of measurement error (data revisions) on macroeconomic activity. Both of these papers use the RBC model and thus do not address monetary issues as we do in this paper. As is typically the case in models with signal extraction, Aruoba finds that it leads to more cautious behaviour, that is inertia.

sions. Barro and Hercowitz, 1981, and Boschen and Grossman, 1982, tested this interpretation of the model but were unable to find any role for data revisions in the monetary transmission mechanism.

In this section we revisit this issue. We establish two facts. First, that the measurement error in monetary aggregates –the difference between preliminary and revised data– represents indeed a measurement error. That, it has the properties required by the imperfect information rational expectations theory, at least for the post great inflation period, and hence it represents a legitimate measure of misperceived money. And that, it is quantitatively significant. And second, that this error (unperceived money) has mattered for economic activity during the last 25 years. For the pre 1982 period, this conventionally constructed measure of misperceived money may not represent a reliable measure of monetary mis-perceptions because it contains predictable components. For that period, and similarly to Barro and Hercowitz, 1981, and Boschen and Grossman, 1982, we find that this variable does not play a role in the business cycle.

Let us first describe the properties of unperceived money. We have used the quarterly real time data constructed at the Philadelphia FED to compute the measurement error for data for a particular period as reported during that as well as subsequent periods (different vintages). In particular, let $M_{t|t}$ be the monetary aggregate (we use M1) of period t that gets reported in period t and $g_{t|t} = logM_{t|t} - logM_{t-1|t}$ its growth rate. This is the initial data release. Let $M_{t|t+i}$ (resp. $g_{t|t+i} = logM_{t|t+i} - logM_{t-1|t+i}$) be the revised figure for period t that is available in period $t + i, i > 0$. We use $t + i = T$ to represent the "final" release[7]. "Unperceived" money growth in t is thus defined as $\mu_{t|T} = g_{t|T} - g_{t|t}$.

Table 1 reports the properties (standard deviation and autocorrelation) of unperceived money growth, and also of unanticipated money shocks, ε_t. Unanticipated shocks correspond to the monetary shock obtained from a VAR model featuring output growth, inflation, the federal fund rate and money supply growth, in that specific order. The monetary shock is identified using a Choleski decomposition of the covariance matrix of the residuals. Unanticipated money shocks are assumed to be the only one that exert a contemporaneous effect on money growth.[8]

Unanticipated shocks are computed based on final data. In order to gain some idea about the quantitative significance of successive revisions of the preliminary data we also report the properties of $\mu_{t|t+i} = g_{t|t+i} - g_{t|t}$ for $i = 1, 2, 4, 8$.

[7]All data is seasonally unadjusted. Kavajecz and Collins, 1995, have argued that the finding of Mankiw et al., 1984, that the Federal Reserve's preliminary estimates of growth rates of the money stock are not unbiased predictors of the growth rates of finally-revised data may arise from the specific seasonal adjustment procedure used by the Federal Reserve.

[8]We also considered an alternative measure of unanticipated shocks that corresponds to the residuals from an autoregressive process for money growth. The results are very similar under the AR(1) and VAR specifications.

Table 1: Properties of misperceived and unanticipated money growth shocks

	σ	ρ	$\rho(\cdot, \varepsilon_t)$
1966Q1–2000Q4			
$\mu_{t,t+1}$	0.12	-0.06	0.10
$\mu_{t,t+2}$	0.17	0.08	0.02
$\mu_{t,t+4}$	0.28	0.20	0.03
$\mu_{t,t+8}$	0.34	0.10	-0.06
$\mu_{t,T}$	0.41	0.05	-0.10
ε_t	0.70	0.01	1.00
1966Q1–1982Q3			
$\mu_{t,t+1}$	0.14	-0.07	-0.05
$\mu_{t,t+2}$	0.20	0.10	-0.20
$\mu_{t,t+4}$	0.34	0.36	-0.12
$\mu_{t,t+8}$	0.40	0.20	-0.07
$\mu_{t,T}$	0.53	0.10	-0.20
ε_t	0.53	-0.07	1.00
1982Q4–2000Q4			
$\mu_{t,t+1}$	0.11	-0.02	0.12
$\mu_{t,t+2}$	0.14	0.07	0.17
$\mu_{t,t+4}$	0.22	-0.18	0.24
$\mu_{t,t+8}$	0.25	-0.16	0.13
$\mu_{t,T}$	0.25	-0.18	0.12
ε_t	0.61	0.03	1.00

Note: σ, ρ and $\rho(\cdot, \varepsilon_t)$ are the standard deviation, 1st order autocorrelation and correlation between unperceived and unanticipated money respectively.

As can be seen, the measurement errors are substantial, with a standard deviation that is about half the size of that of unanticipated money for the whole sample. In the first sub–sample, the measurement error is about the same size as the unanticipated shocks, while it has about one third of the volatility of unanticipated money during the second sub–sample. Overall the volatility of unanticipated shocks has increased over time, while that of measurement error has declined.

We view these findings as establishing that the so constructed "misperceived" money is quantitatively important. It remains to show that this measure of misperceived money corresponds indeed to the concept of monetary mis-perceptions in the model of Lucas. We do so by relying on the approach pioneered by Mankiw et al., 1984. First, we establish that the preliminary announcements of the money stock are best characterized as measured with classical errors-in-variables. This justifies the signal extraction specification employed in the model of the following section. And second, we show that the difference between the initial and the revised announcements *cannot* be predicted on the basis of information that is available at the time of the initial release. Both of these patterns apply to the post but not to the pre 1982 period. Note that the existence of predictability contaminates the conventional measure of misperceived money with anticipated money and renders this variable unsuitable for testing the imperfect information, rational expectations theory of Lucas.

Following Mankiw et al., 1984 we deal with the errors–in–variables issue by regressing the initial release of money growth on a constant term and the final release

$$g_{t|t} = \alpha_0 + \alpha_1 g_{t|T} + u_t$$

and testing the joint null hypothesis, $E\widehat{\alpha}_0 = 0$ and $E\widehat{\alpha}_1 = 1$. The results, as reported in Table 2, indicate that over the second sub–sample, the initial release indeed corresponds to an errors–in–variables phenomenon.

Table 2: Errors-in-Variables

	1966Q1–2000Q4	*1966Q1–1982Q3*	*1982Q4–2000Q4*
F–stat	3.9740	3.2707	1.0211
p–value	[0.0210]	[0.0446]	[0.3656]

In Table 3, we address the issue of the predictability of these errors by regressing $\mu_{t|T}$ on[9] values of the federal fund rate (R) and changes in the stock market (ΔSP) that were available at the time of the release (as in Mankiw et al., 1984). As can be seen from the table, measurement

[9]Note that Table 1 already indicates the absence of autocorrelation and hence predictability based on own lagged values in $\mu_{t|T}$. But this is not sufficient to establish the lack of predictability as there may be other variables at the time of the release that could help forecast future unperceived money shocks.

errors cannot be predicted in the post 1982 but contain predictable elements in the pre-1982 period[10]. This implies that the conventional measure of misperceived money is appropriate for the post 1982 period but may be not for the pre-1982 period as it contains a predictable component.

Table 3: Forecasting regressions

Cst.	R_t	$\Delta S\&P$	D.W.	R^2
1966Q1–2000Q4				
-0.1044	0.1892	1.3396	1.89	0.04
(0.0919)	(4.6836)	(0.5846)		
-0.0460	-2.6123	–	1.90	0.00
(0.0897)	(4.5908)			
-0.1010	–	1.3334	1.89	0.04
(0.0346)		(0.5623)		
1966Q1–1982Q3				
-0.2157	5.3358	2.3233	1.72	0.07
(0.1520)	(7.1656)	(1.0895)		
-0.1620	0.7353	–	1.80	0.00
(0.1540)	(7.0199)			
-0.1136	–	2.0790	1.70	0.06
(0.0655)		(1.0353)		
1982Q4–2000Q4				
0.0093	-2.7307	-0.0020	2.27	0.00
(0.1036)	(6.0738)	(0.5673)		
0.0092	-2.7308	–	2.27	0.00
(0.1018)	(6.0302)			
-0.0347	–	-0.0036	2.26	0.00
(0.0340)		(0.5641)		

Note: R =Federal fund rate, $\Delta S\&P$ = changes in the $S\&P$ stock market index. Standard deviations in parenthesis.

We now turn to the question of whether these measurements errors matter for macroeconomic activity. We have used two alternative methodologies for assessing this issue. One follows Boschen and Grossman, 1982 and involves a regression of the growth rate of output in period t on its lagged values as well as on unperceived money growth during that and previous periods. The other method relies on a standard VAR approach.

We have estimated equations for HP–filtered output (GDP) and the inflation rate (GDP deflator) according to the specification

$$x_t = \sum_{i=1}^{p} \rho_i x_{t-i} + \sum_{\ell=0}^{n} [\alpha_i \mu_{t-\ell} + \beta_i \varepsilon_{t-\ell}] + u_t \quad (1)$$

The unanticipated money shocks $\varepsilon_{t-\ell}$ have been included in the regressions along side the un-

[10]The latter result is identical to that reported by Mankiw et al., 1984.

Table 4: The effects of unperceived and unanticipated money, F–Tests

	Output			Inflation Rate		
	(p,ℓ)	$\mu_{t\vert T}$	ε_t	(p,ℓ)	$\mu_{t\vert T}$	ε_t
1966Q1–2000Q4	(3,2)	3.5139	–	(4,0)	4.5497	–
		[0.0172]			[0.0348]	
1966Q1–1982Q3	(1,2)	2.6078	–	(1,1)	0.7605	–
		[0.0603]			[0.4720]	
1982Q4–2000Q4	(3,0)	3.2369	–	(3,2)	0.4595	–
		[0.0766]			[0.7116]	
1966Q1–2000Q4	(3,2)	–	2.2682	(4,0)	–	0.4920
			[0.0838]			[0.4843]
1966Q1–1982Q3	(1,2)	–	6.3774	(1,1)	–	3.8665
			[0.0008]			[0.0264]
1982Q4–2000Q4	(3,0)	–	0.8553	(3,2)	–	0.2800
			[0.3584]			[0.8396]
1966Q1–2000Q4	(3,2)	3.0166	1.8116	(4,0)	4.2524	0.2346
		[0.0325]	[0.1485]		[0.0413]	[0.6290]
1966Q1–1982Q3	(1,2)	2.0029	5.4925	(1,1)	0.8796	3.8952
		[0.1245]	[0.0023]		[0.4205]	[0.0260]
1982Q4–2000Q4	(3,0)	3.6306	1.2698	(3,2)	0.5232	0.3519
		[0.0612]	[0.2639]		[0.6681]	[0.7879]

Note: p–values in brackets (they correspond to the F-test of the significance of each type of shock). (p,ℓ) refers to the number of lags of the endogenous variable, p, and of the monetary shocks, ℓ. μ_t is unperceived and ε_t is the unanticipated money shock.

perceived one[11] to allow us to judge the relative importance of the two sources of monetary non-neutralities: One arising from nominal rigidities (unanticipated shocks). And the other from informational frictions (unperceived shocks). We test for the significance of unperceived and unanticipated shocks using an F–test. The data start in 1966Q1, the earliest date available in the real data series constructed by the Philadelphia FED. They end in 2000Q4 in order to leave room for the computation of subsequent revisions. We report results for the whole sample and also for the periods 1966–1982 and 1982–2000 separately in order to compare our findings to those of Boschen and Grossman[12]. The number of lags is selected based on the AIC and SC information criterion but the results are robust to using different lag structures. The results are reported in Table 4.

There are two main findings. First, both sources of errors seem to matter for economic activity. And second, unanticipated shocks have had a significant influence on output in the more distant past and unperceived shocks in the more recent past. The former part of the previous statement is in line with the results reported in Barro and Rush for the effects of unanticipated money on

[11]For have simplified notation by using $\mu_t = \mu_{t|T}$. Consequently, $\mu_{t-\ell}$ is unperceived money growth during period $t-\ell$.

[12]Similar results obtain when we use 1979 as the cut off point.

Table 5: The effects of filtered unperceived money, F–Tests

	Output			Inflation Rate		
	(p,ℓ)	μ_t	ε_t	(p,ℓ)	$\mu_{t\|T}$	ε_t
1966Q1-2000Q4	(3,2)	4.9199	–	(4,4)	3.9125	–
		[0.0029]			[0.0025]	
1966Q1-1982Q3	(3,2)	3.3753	–	(4,1)	1.6381	–
		[0.0248]			[0.2040]	
1966Q1-2000Q4	(3,2)	4.2823	1.7224	(4,4)	2.9800	2.2812
		[0.0065]	[0.1658]		[0.0143]	[0.0509]
1966Q1-1982Q3	(3,2)	2.3399	4.7244	(4,1)	1.5112	3.9263
		[0.0843]	[0.0055]		[0.2304]	[0.0259]

Note: p–values in brackets (they correspond to the F-test of the significance of each type of shock). (p,ℓ) refers to the number of lags of the endogenous variable, p, and of the monetary shocks, ℓ. ε_t is the unanticipated money shock. μ_t is unperceived money shock constructed by filtering out the predictable components of the future revisions as of the time of the initial release.

economic activity. The latter part of the statement, namely that measurements errors did not have a statistically significant effect on real economic activity in the early period is in line with with the results reported by of Barro and Hercowitz, 1981, and Boschen and Grossman, 1982.

Can one account for this change in the effects of misperceived money over time? A possible explanation is the one offered by Mankiw et al., 1984, in their evaluation of the Barro-Hercowitz and Boschen-Grossman, results. Namely, the existence of predictability in the money stock revisions that is present in the pre 1982 period (see Table 3 and also Mankiw et al., 1984) introduces an anticipated component into the conventional measure of misperceived money. To the extent that such predictable components are not filtered out, the estimated effect of "misperceived" money on economic activity is biased towards zero.

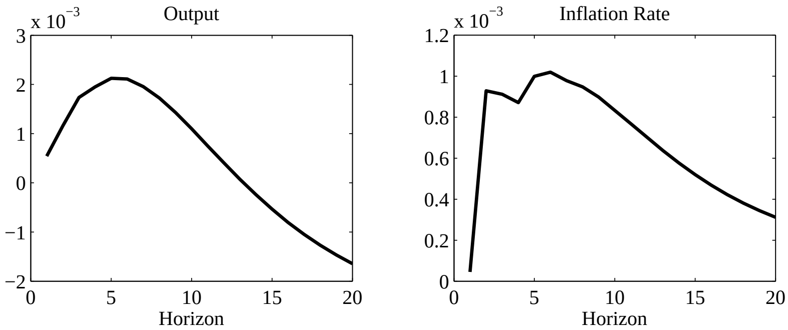
In order to address this issue we proceed to purge the predictable component from the money revisions in the pre 1982 period and use the filtered series to represent unperceived money. Table 5 shows that "properly" constructed unperceived money does matter for economic activity also in the pre 1982 period.

The second method for evaluating the role of measurement error in monetary aggregates relies on VARs of the type that are commonly used in the literature to assess the effects of monetary policy. We have run two VARs. One considers unanticipated (based on final data) money shocks and uses a specification similar to that of CEE, 2005. In particular, we estimate a VAR for money growth, output growth, CPI inflation and the federal fund rate. Standard likelihood ratio tests and information criteria favor the use of a VAR(2) representation. Money appears first in the identification scheme but the results are robust to alternative orderings in the VAR. The other VAR considers unperceived money and uses the μ series described above. In this case, as unperceived money growth ought to be unexplained by any of the other variables in the

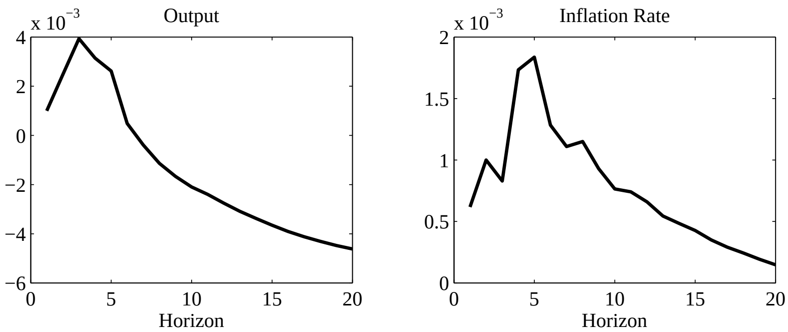
VAR, we estimate a VARX for output growth, CPI inflation and the federal fund rate where μ_t is introduced as an exogenous variable. Standard likelihood ratio tests and information criteria recommend the use of three lags in the VAR part and the current value and three lags of the unperceived money growth series.

As can be seen, from Figure 1 the reaction of output and inflation to a shock, whether unanticipated or misperceived is quite similar. Both output and inflation follow an in increasing path displaying a hump shaped pattern. The effects of misperceived are quantitatively larger.

Figure 1: Response to shocks
(a) unanticipated shock



(b) unperceived shock



2 The model

The previous chapter established empirically that both misperceived and unanticipated money seem to play a significant role in the monetary transmission mechanism. The objective of this section is to examine the role within the NK model. In particular, we argue that both features are critical for obtaining realistic inflation and output dynamics –as well as a liquidity effect– in the NK model. And that in their presence, the NK model has no need for other popular schemes such as myopia (Gali and Gertler, 1999) or backward indexation (Christiano et al., 2005) in order to perform well along this critical dimension.

The basic set up is the NK model with price rigidities, augmented to include various real rigidities. The production side of the economy consists of two sectors: one producing intermediate goods and the other a final good. The intermediate good is produced with capital and labor and the final good with intermediate goods. The final good is homogeneous and can be used for consumption (private and public) and investment purposes.

2.1 Final sector

The final good, y, is produced by combining intermediate goods, y_i, by perfectly competitive firms. The production function is given by

$$y_t = \left(\int_0^1 y_{it}^{\theta} \mathrm{d}i \right)^{\frac{1}{\theta}} \tag{2}$$

where $\theta \in (-\infty, 1)$.

The final good may be used for consumption — private or public — and investment purposes.

2.2 Intermediate goods producers

Each firm i, $i \in (0, 1)$, produces an intermediate good using of capital and labor according to a Cobb–Douglas production function

$$y_{it} = a_t (u_{it} k_{it})^{\alpha} n_{it}^{1-\alpha} \text{ with } \alpha \in (0, 1) \tag{3}$$

where k_{it} and n_{it} are physical capital and labor used by firm i. a_t is an exogenous, stationary, stochastic, technology shock. u_{it} is the rate of capital utilization.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. Following Calvo, we assume that in each and every period, a firm either gets the chance to adjust its price (with probability γ) or it does not. If it does not get the chance, then it sets its price according to

$$P_{it} = \xi_t P_{it-1} \tag{4}$$

We consider two scenarios regarding ξ_t. In the first one, which will be used in the version of the model with the signal extraction formulation, the price is assumed to remain fixed until the firm gets a call that allows it to reset its price optimally. In this case, we have $\xi_t = 1$. In our view, this is the more realistic scenario as the evidence on price setting suggests that firms set their prices infrequently and discretely. The second scenario has these firms index their prices to the lagged, economy wide rate of inflation (as in CEE). Hence $\xi_t = \pi_{t-1}$. As shown by Collard and Dellas, 2005, this assumption plays a critical role in allowing the NK model to produce satisfactory inflation dynamics.

For a firm i that sets its price optimally in period t, its price, $P_t^\star$, is given by

$$P_t^\star = \frac{1}{\theta}\frac{\mathbb{E}_t \sum_{\tau=0}^{\infty}(1-\gamma)^\tau \Phi_{t+\tau} P_{t+\tau}^{\frac{2-\theta}{1-\theta}} \Xi_{t,\tau}^{\frac{1}{\theta-1}} \psi_{t+\tau} y_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0}^{\infty}(1-\gamma)^\tau \Phi_{t+\tau} \Xi_{t,\tau}^{\frac{\theta}{\theta-1}} P_{t+\tau}^{\frac{1}{\theta-1}} y_{t+\tau}} \tag{5}$$

where ψ is real marginal cost, P is the aggregate price index, $\Phi_{t+\tau}$ is an appropriate discount factor derived from the household's optimality conditions and

$$\Xi_{t+\tau} = \begin{cases} \prod_{\ell=0}^{\tau-1} \xi_{t+\ell} & \text{for } \tau \geqslant 1 \\ 1 & \tau = 0 \end{cases}$$

Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price.

2.3 The Household

The preferences of the representative household are given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[\log(c_{t+\tau} - \vartheta c_{t+\tau-1}) + \frac{\nu^m}{1-\sigma_m}\left(\frac{M_{t+\tau}}{P_{t+\tau}}\right)^{1-\sigma_m} - \frac{\nu^h}{1+\sigma_h} h_{t+\tau}^{1+\sigma_h}\right] \tag{6}$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t, M_t/P_t is real balances and h_t is the quantity of labor she supplies. Preferences are characterized by habit persistence governed by the parameter ϑ.

In each period, the representative household faces the budget constraint

$$E_t Q_t B_t + M_t + P_t(c_t + i_t + a(u_t)k_t) = B_{t-1} + M_{t-1} + P_t z_t u_t k_t + P_t w_t h_t + \Omega_t + \Pi_t \tag{7}$$

where B_t is a state contingent claim with corresponding price Q_t. M_t is end of period t money holdings. P_t, the nominal price of goods. c_t and i_t are consumption and investment expenditure respectively; k_t is the amount of physical capital owned by the household and leased to the firms at the real rental rate z_t. Only a fraction u_t of the capital stock is utilized in any period, which involves an increasing and convex cost $a(u_t)$. w_t is the real wage. Ω_t is a nominal lump-sum transfer received from the monetary authority and Π_t denotes the profits distributed to the household by the firms.

Capital accumulates according to the law of motion

$$k_{t+1} = \Phi(i_t, i_{t-1}, k_t) + (1-\delta)k_t \tag{8}$$

where $\delta \in [0, 1]$ denotes the rate of depreciation. $\Phi(\cdot)$ is a general specification that allows the modeling of either capital or investment adjustment costs (its properties will be discussed later).

2.4 The monetary authorities

We assume that monetary policy involves an exogenous money supply rule, in particular the growth rate of the money supply is assumed to follow an exogenous stochastic process. We have also repeated the analysis under a standard interest rate policy rule without any change in the results (see discussion in the results section).

2.5 The government

The government purchases the domestic final good using lump sum taxes. The stationary component of government expenditure, G_t is assumed to follow an exogenous stochastic process whose properties will be defined later.

3 Parametrization

For comparison purposes, the parametrization of the model relies heavily on CEE, 2005. The model is parameterized on US quarterly data for the post WWII period. When necessary, the data are taken from the Federal Reserve Database.[13] The parameters are reported in Table 6.

Table 6: Calibration: Benchmark case

Discount factor	β	0.988
Habit persistence	ϑ	0.650
Inverse labor supply elasticity	σ_h	1.000
Money demand elasticity	σ_m	10.500
Capital elasticity of intermediate output	α	0.281
Parameter of markup	θ	0.850
Depreciation rate	δ	0.025
Adjustment costs parameter	φ	0.330
Probability of price resetting	γ	0.250
Steady state money supply growth (gross)	μ	1.000
Share of government spending	G/y	0.200

[13]URL:http://research.stlouisfed.org/fred/

The capital accumulation function $\Phi(i_t, i_{t-1}, k_t)$ is assumed to take the following form

$$\Phi(i_t, i_{t-1}, k_t) = \left(1 - \omega S\left(\frac{i_t}{i_{t-1}}\right) - (1-\omega)\frac{\varphi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 \frac{k_t}{i_t}\right) i_t$$

The function $S(\cdot)$ satisfies $S(1) = S'(1) = 0$ and $S''(1) = \varphi > 0$. $\Phi(i_t, i_{t-1}, k_t)$ nests two investment adjustment costs ($\omega = 1$) and capital adjustment costs ($\omega = 0$). We mainly focus on the investment adjustment costs case and therefore set $\omega = 1$. The investment adjustment cost parameter φ is then chosen so that the model can match the first order autocorrelation of output (0.84). This implies $\varphi = 0.33$. Note, however, that the same results obtain when we borrow the value of φ used in CEE ($\varphi = 2.5$), instead of calibrating it. The capital utilization function $a(u_t)$ satisfies $a(1) = 0$, $a''(1)/a'(1) = 1/\sigma_a$. We set $\sigma_a = 100$.

The three shocks, the technology shock, $a_t = \log(A_t/\overline{A})$, the fiscal shock, G_t, and the money supply shock are assumed to follow independent, AR(1) processes with persistence parameters ρ_a, ρ_G, ρ_μ respectively and standard deviation of innovations $\sigma_a, \sigma_G, \sigma_\mu$ respectively. These values are given in table 7. The process for government expenditures was estimated on historical data. σ_a was selected so that the model matches the volatility of output (1.49) and σ_μ in order to match the volatility of inflation (0.16) in the model with backward looking price indexation.

Table 7: Shocks

	ρ	σ
Technology	0.9500	0.0042
Fiscal	0.9684	0.0104
Money supply	0.5000	0.0017

3.1 Information

We now specify the structure of information in the case of a signal extraction problem. We assume that while the agents may observe individual specific variables (such as their own consumption, technology shock, capital stock and so on) they can only imperfectly estimate the *true* aggregate state of the economy. Moreover, we assume that the agents learn gradually about the true state using the Kalman filter, based on a set of signals on aggregate variables. Without loss of generality we can assume that some of the aggregate variables may be perfectly observed, some other may not be observed at all and yet some other may be observed with error. For mis–measured variable x we assume that

$$x_t^\star = x_t^{\mathrm{T}} + \eta_t$$

where x_t^{T} denotes the true value of the variable and η_t is a noisy process that satisfies $E(\eta_t) = 0$ for all t; $E(\eta_t \varepsilon_{a,t}) = E(\eta_t \varepsilon_{g,t}) = E(\eta_t \varepsilon_{\mu,t}) = 0$; and

$$E(\eta_t \eta_k) = \begin{cases} \sigma_\eta^2 & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

This specification is consistent with the results reported above regarding the errors-in-variables properties of the money stock announcements.

Knowledge of the *aggregate* state of the economy matters for the agents because individual price setting depends on expectations of future nominal marginal cost and marginal revenue, which in turn depend on future aggregate prices, wages and so on.

An important principle is that the informational constraints are sensible in terms of location, timing and amount of noise. Recall that the objective of our paper is to examine the effects of a monetary policy shock. We cannot allow the *true* value of this shock to be perfectly observable as this does away with the signal extraction problem. But we cannot assume either (without straining credibility) that the agents do not observe monetary aggregates at all (or that they do so with substantial time lags), a common feature of the early vintage of the flexible price, rational expectations models. Based on the findings of the previous section we assume that while information on monetary aggregates is readily available, observations of the current or recent monetary data (the preliminary figures) are ridden with measurement error. This error is corrected through subsequent data revisions.

We assume that the agents receive noisy signals on the variables[14], $\{R_t, \pi, \mu\}$, and in particular on the vector $\{R_t, \pi_t, \pi_{t-1}, \pi_{t-2}, \mu_t, \mu_{t-1}, \mu_{t-2}\}$. The use of lags is motivated by the observation that the initial data announcements are subsequently , periodically revised. We calibrated the variance of the noise on $\{R_t, \pi_t, \pi_{t-1}, \pi_{t-2}, \mu_t, \mu_{t-1}, \mu_{t-2}\}$ by matching the first eight periods in the IRF of inflation to a money shock in the CEE model. The model is thus, *by construction*, able to generate inertial behavior in inflation comparable to that generated in that model. Consequently, its plausibility cannot be accessed by whether the IRFs have the right shape. *It can be assessed by checking whether the amount of required noise is realistic and its location plausible*[15] *and also whether the implications of the model for the other variables is satisfactory.* The calibrated values for the volatility of noise appear in table 8.

As can be seen, the model under imperfect information and learning (signal extraction) requires negligible noise —compared to the volatility of the shocks— on nominal interest rate observa-

[14]Note the the choice of the noisy variables is not restrictive. We could add any other variables to the list without affecting materially the results as long as we satisfied the requirement that signal extraction remained a problem in the model.

[15]This is an important requirement that cannot be evaded by the use of lags or the choice of the noisy variables. There are not effectively any degrees of freedom in the model.

Table 8: Volatility of noise

R_t	π_t	π_{t-1}	π_{t-2}	μ_t	μ_{t-1}	μ_{t-2}
2.23045e-4	3.1301e-3	1.5707e-3	7.8817e-4	8.2173e-3	4.1161e-3	2.0618e-3

tions[16]. The amount of noise on inflation and money supply is also quite small[17]. Note, that it is smaller than that typically used in models of learning in the literature (see, for instance, Woodford, 2002). And it is also within the range implied by our analysis of money revisions above[18].

Note that there is an obvious connection between misperceptions and measurement errors in our model to informational stickiness in the sticky information model. The main difference is that the degree of information stickiness in our specification is imposed by "technology" (the quality of the statistical output), rather than selected by the agents. And that the agents are allowed to learn in a more continuous manner. But both approaches essentially involve agents making decisions based on potentially misperceived states. Our findings above regarding the degree of noise then also suggest that the required degree of information stickiness is also quite small.

4 The results

The model is log–linearized around its deterministic steady state and then solved. The solution method for the case in which the agents solve a signal extraction problem is to be found in a technical appendix available from the authors' web pages.

Figure 2 presents the response of inflation, output, the nominal and the real interest rate to a 1% shock to the growth rate of the money supply under three model specifications: *(i)* The standard, forward looking, NK model (forward looking); *(ii)* the version with indexation (backward looking); and *(iii)* the forward looking version with signal extraction. In *all three cases*, the model includes three real rigidities, namely, habit persistence, variable capital utilization and investment adjustment costs.[19].

[16]Implementing monetary policy with an interest rate rule that includes a policy shock allows the model to operate well with zero noise on R; see below.

[17]This specification represents a critical departure from Dellas, 2006. Dellas demonstrates that the NK model with a signal extraction problem *may* generate persistence in inflation and output. But the key question is not whether such a model can generate inflation inertia (this is already hinted in other related work, such as Svensson and Woodford, 2003) but whether it can do so under plausible informational assumptions. Dellas' example requires a large amount of noise on all nominal variables, including R.

[18]It is also consistent with other estimates. For instance, the BEA reports "preliminary" and revised values for the GDP deflator. The standard deviation of the difference between announced and revised values for the GDP deflator from 1999-2003 was 0.48%.

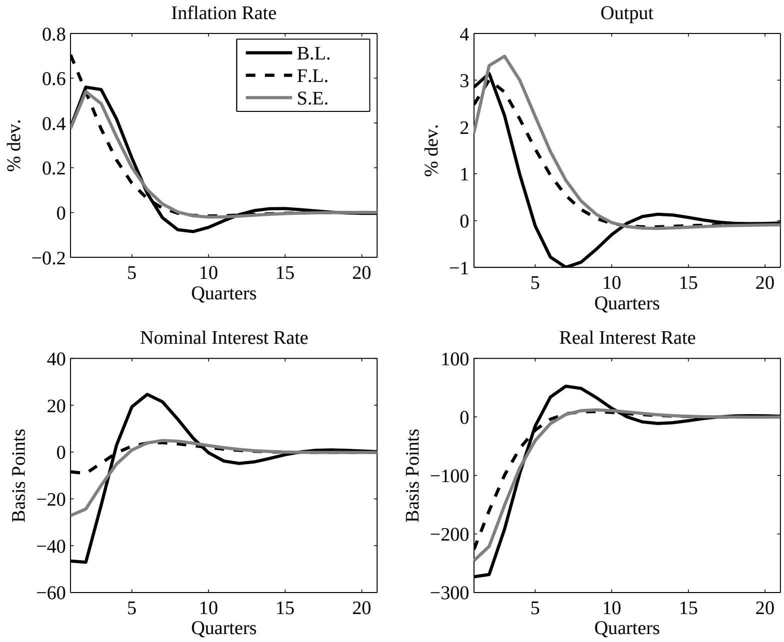
[19]Using capital in place of investment adjustment costs makes no difference for the behavior of the model with signal extraction.

Table 9: HP moments

Var.	Std	Rel. Std	$\rho(\cdot, y)$	$\rho(1)$	$\rho(2)$
		Data			
y	1.49	1.00	1.00	0.88	0.70
c	0.80	0.54	0.86	0.87	0.69
i	6.03	4.04	0.92	0.83	0.61
h	1.88	1.26	0.83	0.92	0.73
π	0.16	0.11	0.32	0.33	0.24
R_{nom}	0.40	0.27	0.21	0.81	0.57
R_{real}	0.33	0.22	0.10	0.73	0.50
		(b) Forward looking (no indexation)			
y	1.35	1.00	1.00	0.87	0.64
c	0.21	0.16	0.86	0.88	0.65
i	4.12	3.06	0.94	0.90	0.70
h	0.83	0.61	0.88	0.76	0.47
π	0.16	0.12	0.61	0.56	0.22
R_{nom}	0.01	0.01	-0.53	0.80	0.45
R_{real}	0.12	0.09	-0.63	0.56	0.21
		(a) Backward looking (indexation)			
y	1.49	1.00	1.00	0.84	0.55
c	0.23	0.16	0.90	0.86	0.60
i	4.49	3.01	0.95	0.88	0.63
h	0.87	0.58	0.90	0.74	0.38
π	0.16	0.11	0.54	0.81	0.43
R_{nom}	0.04	0.03	-0.75	0.74	0.29
R_{real}	0.18	0.12	-0.71	0.70	0.28
		(b) Signal extraction			
y	1.51	1.00	1.00	0.83	0.56
c	0.21	0.14	0.91	0.87	0.62
i	4.19	2.77	0.94	0.89	0.68
h	1.08	0.71	0.91	0.74	0.42
π	0.16	0.11	0.74	0.62	0.27
R_{nom}	0.02	0.02	-0.23	0.67	0.30
R_{real}	0.15	0.10	-0.57	0.66	0.28

Note: All series are HP–filtered. Data cover the period 1960:1–2002:4, except for aggregate weekly hours that run from 1964:1 to 2002:4. Output is defined as $C+I+G$. C is nondurables and services, I includes investment and durables. π is the CPI based inflation rate, R_{nom} is the federal fund rate, and $R_{real} = R_{nom} - \pi$. Std. is standard deviation, Rel. Std is standard deviation of the variable relative to that of output, $\rho(\cdot, y)$ is its correlation with output and $\rho(1)$ and $\rho(2)$ the first and second order autocorrelation.

Figure 2: IRF to a money supply shock



Note: Three model specifications: a) B.L.: Backward looking, b) F.L.: Forward looking, c) S.E.: Signal extraction.

As can be seen, the three versions perform comparably with one important exception. Namely, the response of inflation. The version with forward looking agents cannot generate inflation inertia. This finding confirms the well known fact (see Collard and Dellas, 2005) that price staggering does not suffice to produce plausible dynamics. It also demonstrates that real rigidities alone cannot help the NK model deliver the hump either[20]. For instance, there is a widely held view that habit persistence is sufficient to generate inertial behavior. As Figure 2 shows (see also Collard and Dellas, 2005) this is not the case. On the other hand, it must be emphasized that real rigidities are important in order to generate sufficient inertia under either backward indexation or signal extraction. This is illustrated in in the technical Appendix (available at the authors' web pages) which shuts all real rigidities down.

Naturally, there is a trade off between the degree of measurement error and the strength of real rigidities required to produce inertial behavior. As Dellas, 2006, shows, very persistent, hump shaped responses of the key macroeconomic variables can obtain without the need for any real rigidities when the amount of noise on nominal aggregates is very high. But real rigidities and signal extraction also play distinct roles. Real rigidities alone (in the absence of signal extraction or backward indexation) cannot produce a hump in the dynamics of inflation, irrespective of

[20]Notice that the degree of inflation inertia could be increased further by increasing the amount of real rigidities, by introducing additional, commonly used inertial features such as wage stickiness, expenditure lags and so on. We have not done so because it would not do affect the main point of this paper.

their size, while signal extraction can.

What is the role played by price rigidity? We have also carried out the analysis under flexible prices. Unless one is willing to accept unreasonably high levels of informational frictions (very large noise on monetary aggregates), the model cannot produce plausible inflation dynamics.

Table 9 reports unconditional moments both in the data and under the three model specifications. The performance of the models is comparable. Their main weaknesses are to be found in the under-prediction of volatilities (in particular of consumption and the nominal interest rate) as well as their implication of counter–cyclicality in the interest rates. Note, that the model with signal extraction does somewhat better along the last dimension. Canzoneri et al., 2004, argue that there exists no model that can adequately capture interest rate behavior, so this weakness is not specific to these NK models.

How robust are our findings with regard to the specification of the monetary policy rule? We have repeated the analysis with a standard interest rate (Henderson-McKibbin-Taylor) rule modified to include a variable inflation target (so that there is a monetary policy shock present in the model).

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)\left[\log(\overline{R}) + \kappa_\pi(\log(\pi_t) - \log(\pi_t^\star)) + \kappa_y(\log(y_t) - \log(y^\star))\right]$$

where the output target, $y^\star$, is the steady state level of output. $\pi_t^\star$ is the inflation target and is assumed to follow a random walk.[21] The parameters of the interest rate rule are ρ_r=0.75, κ_y=0.2 and κ_π=1.8. The noise is calibrated as before but now we assume that there is no noise at all in the observations in the interest rate. [22]

Figure 3 shows the IRFs to an inflation target (policy) shock. They are virtually indistinguishable from those in Figure 2.

5 Conclusions

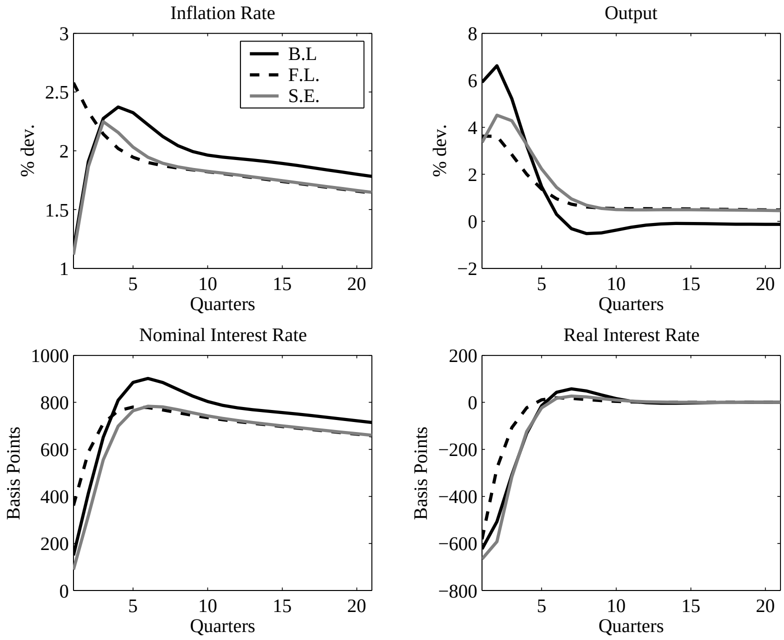
The introduction of the imperfect information, rational expectations paradigm was followed by intensive debate regarding the role of misperceived money (the key ingredient of the flexible price version) relative to that of unanticipated money (the key ingredient of the fixed price version). While this debate was settled at the time conclusively in favor of the unanticipated

[21]In practice, we use an AR(1) process with persistence parameter 0.9999. The standard deviation of the innovation is set such that, borrowing all other parameters from the previous version of the model, the model matches output volatility.

[22]In particular the noise is now given by:

R_t	π_t	π_{t-1}	π_{t-2}	μ_t	μ_{t-1}	μ_{t-2}
0	1.9764e-3	9.8864e-4	4.9455e-4	2.4191e-3	1.5699e-3	1.0187e-3

Figure 3: IRF to an inflation target shock



Note: Three model specifications: a) B.L.: Backward looking, b) F.L.: Forward looking, c) S.E.: Signal extraction.

money version, the old debate seems to have been resurrected in the context of the sticky price vs sticky information models. We have revisited this issue both empirically and theoretically. At the empirical front, we have established that the difference between initial and subsequent releases of the money stock does not always correspond to the theoretically correct measure of misperceived money. When it does, or when the anticipated components it may contain are purged, then it does matter for the business cycle. At the theoretical front, we have established that both unperceived and unanticipated money shocks are sina qua non for the ability of the forward looking new Keynesian model to generate plausible dynamics. And that the amount of misperceptions of monetary aggregates (information stickiness) is small and plausible.

The success of this combination provides welcome relief to the NK model, as its only other means so far of getting plausible dynamics involves reliance on empirically tenuous pricing schemes (such as backward indexation). Our findings reveal the value of a new synthesis between models that emphasize informational and models that emphasize nominal frictions. Both features seem quite essential for a monetary model to exhibit good performance.

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